

ALGORITHM 64

QUICKSORT

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```
procedure quicksort (A,M,N); value M,N;  
    array A; integer M,N;  
comment Quicksort is a very fast and convenient method of  
sorting an array in the random-access store of a computer. The  
entire contents of the store may be sorted, since no extra space is  
required. The average number of comparisons made is  $2(M-N) \ln$   
 $(N-M)$ , and the average number of exchanges is one sixth this  
amount. Suitable refinements of this method will be desirable for  
its implementation on any actual computer;  
begin    integer I,J;  
        if M < N then begin partition (A,M,N,I,J);  
            quicksort (A,M,J);  
            quicksort (A, I, N)  
        end  
end    quicksort
```

ALGORITHM 65

FIND

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```
procedure find (A,M,N,K); value M,N,K;  
    array A; integer M,N,K;  
comment Find will assign to A [K] the value which it would  
have if the array A [M:N] had been sorted. The array A will be  
partly sorted, and subsequent entries will be faster than the first;
```

```

begin      integer I,J;
           if M < N then begin partition (A, M, N, I, J);
                           if K ≤ I then find (A,M,I,K)
                           else if J ≤ K then find (A,J,N,K)
                           end
end        find

```

ALGORITHM 66

INVRs

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procedure InvrS (t) size : (n); **value** n; **real array** t; **integer** n;

comment Inverts a positive definite symmetric matrix t, of order n, by a simplified variant of the square root method. Replaces the $n(n+1)/2$ diagonal and superdiagonal elements of t with elements of t^{-1} , leaving subdiagonal elements unchanged. Advantages: only n temporary storage registers are required, no identity matrix is used, no square roots are computed, only n divisions are performed, and, as n becomes large, the number of multiplications approaches $n^3/2$;

begin integer i, j, s; **real array** v[1:n-1]; **real** y, pivot;

for s := 0 **step** 1 **until** n-1 **do**

begin pivot := 1.0/t[1,1];

begin pivot := 1.0/t[1,1];

comment If $t[1,1] \leq 0$, t is not positive definite;

for i := 2 **step** 1 **until** n **do** v[i-1] := t[1, i];

for i := 1 **step** 1 **until** n-1 **do**

begin t[i,n] := y := -v[i] × pivot;

for j := i **step** 1 **until** n-1 **do**

t[i, j] := t[i + 1, j + 1] + v[j] × y

end;

t[n,n] := -pivot

end;

comment At this point, elements of t^{-1} occupy the original array space but with signs reversed, and the following statements effect a simple reflection;

for i := 1 **step** 1 **until** n **do**

for j := i **step** 1 **until** n **do** t[i,j] := -t[i,j]

end InvrS

ALGORITHM 67

CRAM

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procedure CRAM (n, r, a) Result: (f); **value** n, r; **integer** n, r; **real array** a, f;

comment CRAM stores, via an unspecified input procedure READ, the diagonal and superdiagonal elements of a square symmetric matrix e, of order n, as a pseudo-array of dimension $1:n(n+1)/2$. READ (u) puts one number into u. Elements $e[i, j]$ are addressable as $a[c + j]$, where $c = (2n - i)(i - 1)/2$ and $c[i + 1]$ may be found as $c[i] + n - i$. Since $c[1] = 0$, it is simpler to develop a table of the $c[i]$ by recursion, as shown in the sequence labelled "table". Further manipulation of the elements so stored is illustrated by premultiplying a rectangular matrix f, of order n, r, by the matrix e, replacing the elements of f with the new values, requiring a temporary storage array v of dimension $1:n$;

```

begin integer i, j, k, m; real array v[1:n]; real s;
integer array c[1:n];
table: j := -n; k := n + 1; for i := 1 step 1 until n do
begin
j := j + k - i; c[i] := j end;
load: for i := 1 step 1 until n do
begin for j := i step 1 until n do READ (v[j]); m := c[i];
for k := i step 1 until n do a[m + k] := v[k] end;
premult: for j := 1 step 1 until r do
begin for i := 1 step 1 until n do
begin s := 0.0;
for k := 1 step 1 until i do
begin m := c[k]; s := s + a[m + i]
× f[k, j] end;
for k := i + 1 step 1 until n do
s := s + a[m + k] × f[k, j]; v[i] = s
end;
for k := 1 step 1 until n do f[k, j] = v[k]
end
end
end CRAM

```

REMARK ON ALGORITHM 53

Nth ROOTS OF A COMPLEX NUMBER (John R.

Herndon, *Comm. ACM* 4, Apr. 1961)

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A considerable saving of machine time for $N \geq 3$ would result from the use of the recursion formulas for the sine and cosine in place of an entry into a sine-cosine subroutine in the do loop associated with the Nth roots of a complex number. That is, one could use

$$\sin(n+1)\theta = \sin n\theta \cos\theta + \cos n\theta \sin\theta$$

$$\cos(n+1)\theta = \cos n\theta \cos\theta - \sin n\theta \sin\theta,$$

at the cost of some additional storage.

We have found this procedure to be very efficient in problems dealing with Fourier analysis, as suggested by G. Goerzel in chapter 24 of *Mathematical Methods for Digital Computers*.

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